SOLUTION OF EXERCISE # 5.5

Exercise # 5.5

Solve the triangle ABC with given data.

Q.1: $c = 4$, $\alpha = 70^\circ$, $\gamma = 42^\circ$ (IA-2016)

Sol. Here $a = ?$, $b = ?$, $\beta = ?$

We know that: $\alpha + \beta + \gamma = 180^\circ$

$$\beta = 180^\circ - \alpha - \gamma$$

$$\beta = 180^\circ - 70^\circ - 42^\circ \Rightarrow \boxed{\beta = 68^\circ}$$

By using Law of Sines : $\boxed{\dfrac{a}{\sin\alpha} = \dfrac{b}{\sin\beta} = \dfrac{c}{\sin\gamma}}$

We take $\dfrac{a}{\sin\alpha} = \dfrac{c}{\sin\gamma}$

$$a = \frac{c\sin\alpha}{\sin\gamma}$$

$$a = \frac{4\sin 70^\circ}{\sin 42^\circ} \Rightarrow \boxed{a = 5.62}$$

Again take $\dfrac{b}{\sin\beta} = \dfrac{c}{\sin\gamma}$

$$b = \frac{c\sin\beta}{\sin\gamma}$$

$$b = \frac{4\sin 68^\circ}{\sin 42^\circ} \Rightarrow \boxed{b = 5.54}$$

Q.2: $a = 464$, $\beta = 102^\circ$, $\gamma = 23^\circ$

Sol. Here $b = ?$, $c = ?$, $\alpha = ?$

We know that: $\alpha + \beta + \gamma = 180^\circ$

$$\alpha = 180^\circ - \beta - \gamma \Rightarrow \alpha = 180^\circ - 102^\circ - 23^\circ \Rightarrow \boxed{\alpha = 55^\circ}$$

By using Law of Sines : $\boxed{\dfrac{a}{\sin\alpha} = \dfrac{b}{\sin\beta} = \dfrac{c}{\sin\gamma}}$

We take $\dfrac{b}{\sin\beta} = \dfrac{a}{\sin\alpha}$

$$b = \frac{a\sin\beta}{\sin\alpha}$$

$$b = \frac{464\sin 102^\circ}{\sin 55^\circ}$$

$$\boxed{b = 554.06}$$

Again take $\dfrac{c}{\sin\gamma} = \dfrac{a}{\sin\alpha}$

$$c = \frac{a\sin\gamma}{\sin\alpha}$$

$$c = \frac{464\sin 23^\circ}{\sin 55^\circ}$$

$$\boxed{c = 221.33}$$

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Q.3: $b = 85,\ \beta = 57^\circ 15',\ \gamma = 78^\circ 18'$

Sol. Here $a = ?,\quad c = ?,\quad \alpha = ?$

We know that: $\alpha + \beta + \gamma = 180^\circ$

$$\alpha = 180^\circ - \beta - \gamma$$

$$\alpha = 180^\circ - 57^\circ 15' - 78^\circ 18' \Rightarrow \boxed{\alpha = 44^\circ 27'}$$

By using Law of Sines : $\boxed{\dfrac{a}{\sin\alpha} = \dfrac{b}{\sin\beta} = \dfrac{c}{\sin\gamma}}$

We take $\dfrac{a}{\sin\alpha} = \dfrac{b}{\sin\beta}$

$$a = \frac{b\sin\alpha}{\sin\beta}$$

$$a = \frac{85\sin 44^\circ 27'}{\sin 57^\circ 15'}$$

$$\boxed{a = 70.77}$$

Again take $\dfrac{c}{\sin\gamma} = \dfrac{b}{\sin\beta}$

$$c = \frac{b\sin\gamma}{\sin\beta}$$

$$c = \frac{85\sin 78^\circ 18'}{\sin 57^\circ 15'}$$

$$\boxed{c = 98.97}$$

Q.4: $b = 56.8,\ \alpha = 79^\circ 31',\ \beta = 44^\circ 24'$

Sol. Here $a = ?,\quad c = ?,\quad \gamma = ?$

We know that: $\alpha + \beta + \gamma = 180^\circ$

$$\gamma = 180^\circ - \alpha - \beta$$

$$\gamma = 180^\circ - 79^\circ 31' - 44^\circ 24' \Rightarrow \boxed{\gamma = 56^\circ 5'}$$

By using Law of Sines : $\boxed{\dfrac{a}{\sin\alpha} = \dfrac{b}{\sin\beta} = \dfrac{c}{\sin\gamma}}$

We take $\dfrac{a}{\sin\alpha} = \dfrac{b}{\sin\beta}$

$$a = \frac{b\sin\alpha}{\sin\beta}$$

$$a = \frac{56.8\sin 79^\circ 31'}{\sin 44^\circ 24'}$$

$$\boxed{a = 79.83}$$

Again take $\dfrac{c}{\sin\gamma} = \dfrac{b}{\sin\beta}$

$$c = \frac{b\sin\gamma}{\sin\beta}$$

$$c = \frac{56.8\sin 56^\circ 5'}{\sin 44^\circ 24'}$$

$$\boxed{c = 67.37}$$

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Q.5: $b = 34.57,\ \alpha = 62^\circ 11',\ \beta = 63^\circ 22'$

Sol. Here $a = ?,\quad c = ?,\quad \gamma = ?$

We know that

$$\alpha + \beta + \gamma = 180^\circ$$
$$\gamma = 180^\circ - \alpha - \beta$$
$$\gamma = 180^\circ - 62^\circ 11' - 63^\circ 22'$$
$$\boxed{\gamma = 54^\circ 27'}$$

By using Law of Sines : $\boxed{\dfrac{a}{\sin\alpha} = \dfrac{b}{\sin\beta} = \dfrac{c}{\sin\gamma}}$

We take $\dfrac{a}{\sin\alpha} = \dfrac{b}{\sin\beta}$

$$a = \frac{b\sin\alpha}{\sin\beta}$$
$$a = \frac{34.57\sin 62^\circ 11'}{\sin 63^\circ 22'}$$
$$\boxed{a = 34.20}$$

Again take $\dfrac{c}{\sin\gamma} = \dfrac{b}{\sin\beta}$

$$c = \frac{b\sin\gamma}{\sin\beta}$$
$$c = \frac{(34.57)\sin 54^\circ 27'}{\sin 63^\circ 22'}$$
$$\boxed{c = 31.47}$$

Q.6: Find the angle of largest measure in the triangle ABC where:

(i) $a = 224,\quad b = 380,\quad c = 340$

Sol. As side b is greater so we will find angle β.

Using law of cosines.

$$b^2 = c^2 + a^2 - 2\,ca\cos\beta$$
$$2\,ca\cos\beta = c^2 + a^2 - b^2$$
$$\cos\beta = \frac{c^2 + a^2 - b^2}{2ca} = \frac{(340)^2 + (224)^2 - (380)^2}{2(340)(224)} = 0.1403$$

$$\beta = \cos^{-1}(0.1403) \quad \Rightarrow \quad \boxed{\beta = 81^\circ 55' 57''}$$

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(ii) $a = 374, \quad b = 514, \quad c = 425$

Sol. As side b is greater so we will find angle β.

Using law of cosines:

$$b^2 = c^2 + a^2 - 2ca\cos\beta$$

$$2ca\cos\beta = c^2 + a^2 - b^2$$

$$\cos\beta = \frac{c^2 + a^2 - b^2}{2ca} = \frac{(425)^2 + (374)^2 - (514)^2}{2(425)(374)} = 0.1771$$

$$\beta = \cos^{-1}(0.1771) \quad \Rightarrow \quad \boxed{\beta = 79^\circ 47' 53''}$$

Q.7: Solve the triangle ABC where:

(i) $a = 74, \quad b = 52, \quad c = 47$

Sol. Here $\alpha = ?, \quad \beta = ?, \quad \gamma = ?$

Using law of cosines:

$$a^2 = b^2 + c^2 - 2bc\cos\alpha$$

$$2bc\cos\alpha = b^2 + c^2 - a^2$$

$$\cos\alpha = \frac{b^2 + c^2 - a^2}{2bc}$$

$$\cos\alpha = \frac{(52)^2 + (47)^2 - (74)^2}{2(52)(47)}$$

$$\cos\alpha = -0.1152$$

$$\alpha = \cos^{-1}(-0.1152)$$

$$\boxed{\alpha = 96^\circ 36' 50''}$$

Again using law of cosines:

$$b^2 = c^2 + a^2 - 2ca\cos\beta$$

$$2ca\cos\beta = c^2 + a^2 - b^2$$

$$\cos\beta = \frac{c^2 + a^2 - b^2}{2ca}$$

$$\cos\beta = \frac{(47)^2 + (74)^2 - (52)^2}{2(47)(74)}$$

$$\cos\beta = 0.7161$$

$$\beta = \cos^{-1}(0.7161)$$

$$\boxed{\beta = 44^\circ 16' 7''}$$

As we know that: $\alpha + \beta + \gamma = 180^\circ$

$$\gamma = 180^\circ - \alpha - \beta$$

$$\gamma = 180^\circ - 96^\circ 36' 50'' - 44^\circ 16' 7'' \Rightarrow \boxed{\gamma = 39^\circ 7' 3''}$$

(ii) $a = 7, \quad b = 9, \quad c = 7$

Sol. Here $\alpha = ?, \quad \beta = ?, \quad \gamma = ?$

Using law of cosines:

$$a^2 = b^2 + c^2 - 2bc\cos\alpha$$

$$2bc\cos\alpha = b^2 + c^2 - a^2$$

Again using law of cosines:

$$b^2 = c^2 + a^2 - 2ca\cos\beta$$

$$2ca\cos\beta = c^2 + a^2 - b^2$$

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$$\cos\alpha = \frac{b^2+c^2-a^2}{2bc}$$

$$\cos\alpha = \frac{(9)^2+(7)^2-(7)^2}{2(9)(7)}$$

$$\cos\alpha = 0.6429$$

$$\alpha = \cos^{-1}(0.6429)$$

$$\boxed{\alpha = 50^\circ}$$

$$\cos\beta = \frac{c^2+a^2-b^2}{2ca}$$

$$\cos\beta = \frac{(7)^2+(7)^2-(9)^2}{2(7)(7)}$$

$$\cos\beta = 0.1735$$

$$\beta = \cos^{-1}(0.1735)$$

$$\boxed{\beta = 80^\circ}$$

As we know that : $\alpha+\beta+\gamma = 180^\circ$

$$\gamma = 180^\circ - \alpha - \beta$$

$$\gamma = 180^\circ - 50^\circ - 80^\circ \Rightarrow \boxed{\gamma = 50^\circ}$$

(iii) $\mathbf{a = 2.3,}$ $\mathbf{b = 1.5,}$ $\mathbf{c = 2.7}$

Sol. Here $\alpha = ?,\ \beta = ?,\ \gamma = ?$

Using law of cosines:

$$a^2 = b^2 + c^2 - 2bc\cos\beta$$

$$2bc\cos\alpha = b^2 + c^2 - a^2$$

$$\cos\alpha = \frac{b^2+c^2-a^2}{2bc}$$

$$\cos\alpha = \frac{(1.5)^2+(2.7)^2-(2.3)^2}{2(1.5)(2.7)}$$

$$\cos\alpha = 0.5247$$

$$\alpha = \cos^{-1}(0.5247)$$

$$\boxed{\alpha = 58^\circ 21' 9''}$$

Again by law of cosines:

$$b^2 = c^2 + a^2 - 2ca\cos\beta$$

$$2ca\cos\beta = c^2 + a^2 - b^2$$

$$\cos\beta = \frac{c^2+a^2-b^2}{2ca}$$

$$\cos\beta = \frac{(2.7)^2+(2.3)^2-(1.5)^2}{2(2.7)(2.3)}$$

$$\cos\beta = 0.8317$$

$$\beta = \cos^{-1}(0.8317)$$

$$\boxed{\beta = 33^\circ 43' 25''}$$

As we know: $\alpha+\beta+\gamma = 180^\circ$

$$\gamma = 180^\circ - \alpha - \beta$$

$$\gamma = 180^\circ - 58^\circ 21' 9'' - 33^\circ 43' 25'' \Rightarrow \boxed{\gamma = 87^\circ 55' 26''}$$